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INFINITE FAMILIES OF BASE SYSTEMS AND PROPERTIES OF NUMBERS

R. NARAYANAN

Abstract. In this paper we deal with families of base systems and properties of numbers. Various standard results for base 10 are generalized appropriately for various base systems. The elaborate but elementary proof by the method of mathematical induction on two or three variables is omitted in many cases.

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Introduction

In this paper we deal with families of base systems and properties of numbers. For this we need a consistent notation for the Unit-Integers of any base system B for *All values of* $B \geq 2$.

Let $B \geq 2$ be any given base.

We define the Unit-Integers in this base system thus

$$\left(\boxed{0}, \boxed{1}, \boxed{2}, \boxed{3}, \boxed{4}, \boxed{5}, \cdots, \boxed{\text{B-3}}, \boxed{\text{B-2}}, \boxed{\text{B-1}}.\right)$$

For example let $B = 1729$.
We define the Unit-Integers in this base system thus

$$\left(\boxed{0}, \boxed{1}, \boxed{2}, \boxed{3}, \boxed{4}, \boxed{5}, \cdots, \boxed{1726}, \boxed{1727}, \boxed{1728}.\right)$$

Then the number

$$\boxed{134}\boxed{734}\boxed{894}\boxed{674}\boxed{583}\boxed{13}\boxed{10}_{[1729]}$$
$$= 10.(1729)^0 + 13.(1729)^1 + 583.(1729)^2$$
$$+ 674.(1729)^3 + 894.(1729)^4$$
$$+ 734.(1729)^5 + 134.(1729)^6$$

in the decimal system.

Theorem 1. Let $N = a_m B^m + a_{m-1} B^{m-1} + \cdots + a_1 B + a_0$ be the representation of the positive integer N to the base B. $0 \leq a_k < B$, and let $R = a_0 - a_1 + a_2 - \cdots + (-1)^m a_m$.
Then $(B+1)|N$ if and only if $(B+1)|R$.

Proof. Let $\mathfrak{P}(x) = \sum_{k=0} a_k x^k$ be a polynomial with Integer Coefficients.
Since $B \equiv -1(\mathrm{mod}(B+1))$, we get $\mathfrak{P}(B) \equiv \mathfrak{P}(-1)(\mathrm{mod}(B+1))$.
But $\mathfrak{P}(B) \equiv N$, whereas $\mathfrak{P}(-1) = a_0 - a_1 + a_2 - \cdots + (-1)^m a_m = R$, so that $N \equiv R(\mathrm{mod}(B+1))$. This implies that either both N and R are divisible by $(B+1)$ or neither is divisible by $(B+1)$.
(For $B = 10$ we have the standard result.) [**1**, p. 93] □

Theorem 2. Let $N = a_m 10^m + a_{m-1} 10^{m-1} + \cdots + a_1 10 + a_0$ be the Decimal Expansion of the positive integer N, $0 \leq a_k < 10$, and let $J = a_0 + a_1 + a_2 + \cdots + a_m$.
Then $10^\xi - 1 | N$ if and only if $9\xi | J$. $[\xi = 1, 2, \cdots]$

Proof. For $\xi = 1$ we have the standard result [**1**, p. 93]. Prove by the application of the principle of mathematical induction on the variable "ξ". □

Theorem 3. Let $N = a_m B^m + a_{m-1} B^{m-1} + \cdots + a_1 B + a_0$ be the representation of the positive integer N to the base B, $0 \leq a_k < B$, and let $J = a_0 + a_1 + a_2 + \cdots + a_m$.
Then $(B^\xi - 1) | N$ if and only if $\xi(B-1) | J$. $[\xi = 1, 2, \cdots]$

Proof. Prove by the application of the principle of mathematical induction on the variable "ξ". □

Theorem 4. Let $F = \{B = 10\xi \ [\xi = 1, 2, \cdots]\}$ be a family of base systems.

Then for every integer "a" the unit digit of a^2 is

$$\boxed{10\delta}, \boxed{10\delta + 1}, \boxed{10\delta + 4}, \boxed{10\delta + 5}, \boxed{10\delta + 6}$$
$$\text{or } \boxed{10\delta + 9}. \; [\delta = 0, 1, 2, \cdots (\xi - 1)]$$

in each member system in this family of base systems.

Proof. For $\xi = 1$ we have the standard result [**1**, p. 94]. Prove by the application of the principle of mathematical induction on the variable "ξ". □

Theorem 5. Let $F = \{B\ [B = 2, 3, \cdots]\}$ be a family of base systems. For every integer "a" the unit digit of a^3 can be any integer $\boxed{x} < B$ where B is the base of the system of notation.

Proof. For $B = 10$ we have the standard result [**1**, p. 94]. Prove by the application of the principle of mathematical induction on the variable "B". □

Theorem 6. Let $F = \{B\ [B = 2, 3, \cdots]\}$ be a family of base systems. For every integer "a" the unit digit of a^τ [τ any odd number] can be any integer $\boxed{x} < B$ where B is the base of the system of notation.

Proof. For $B = 10$ and $\tau = 3$ we have the standard result [**1**, p. 94]. Prove by the application of the principle of mathematical induction on the variable "B" and "τ". □

Theorem 7. Let $F = \{B = 10\xi\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.

Then for every integer “a” the unit digit of a^4 is

$$\boxed{10\delta}, \boxed{10\delta+1}, \boxed{10\delta+5}, \text{ or } \boxed{10\delta+6}. \ [\delta = 0, 1, 2, \cdots (\xi - 1)]$$

in each member system in this family of base systems.

Proof. For $\xi = 1$ we have the standard result [**1**, p. 94]. Prove by the application of the principle of mathematical induction on the variable “ξ”. □

Theorem 8. Let $F = \{B = 10\xi \ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then for every integer “a” the unit digit of $a^{4\tau}$ $[\tau \geq 1]$ is

$$\boxed{10\delta}, \boxed{10\delta+1}, \boxed{10\delta+5} \text{ or } \boxed{10\delta+6}. \ [\delta = 0, 1, 2, \cdots (\xi - 1)]$$

in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variables “ξ” and “τ”. □

Theorem 9. Let $F = \{B = 10\xi \ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then the unit digit of a Triangular a Number is

$$\boxed{10\delta}, \boxed{10\delta+1}, \boxed{10\delta+3}, \boxed{10\delta+5}, \boxed{10\delta+6} \text{ or } \boxed{10\delta+8}. \ [\delta = 0, 1, 2, \cdots (\xi - 1)]$$

in each member system in this family of base systems.

Proof. For $\xi = 1$ we have the standard result [**1**, p. 94]. Prove by the application of the principle of mathematical induction on the variable "ξ". $\square$

Theorem 10. Let $F = \{B = 2\xi\ [\xi = 1, 2, \cdots]\}$ be a family of base systems. Then any integer is divisible by $\boxed{2}$ if and only if its units digit is $\boxed{2\delta}$ $[\delta = 0, 1, 2, \cdots\ (\xi - 1)]$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variable "ξ". (For $\xi = 5$ we have the standard result [**1**, p. 94].) $\square$

Theorem 11. Let $F = \{B = 3\xi + 1\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then any integer is divisible by $\boxed{\xi}$ if and only if the sum of its digits is divisible by $\boxed{\xi}$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variable "ξ". (For $\xi = 3$ we have the standard result [**1**, p. 94].) $\square$

Theorem 12. Let $F = \{B = 3\xi + 1\ [\xi = 3, 4, \cdots]\}$ be a family of base systems.
Then any integer is divisible by the integer $\boxed{3}$ if and only if the sum of its digits is divisible by $\boxed{3}$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variable "ξ". (For $\xi = 3$ we have the standard result [**1**, p. 94].) □

Theorem 13. Let $F = \{B = 10\xi\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then any integer of three or more digits is divisible by the integer $\boxed{4\delta}$ if and only if the number formed by its tens and units digits is divisible by $\boxed{4\delta}$ $[\delta = 1, 2, \cdots \xi]$ in each member system in this family of base systems.

Proof. For $\xi = 1$ we have the standard result [**1**, p. 94]. Prove by the application of the principle of mathematical induction on the variable "ξ". □

Theorem 14. Let $F = \{B = 10\xi\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then any integer of three or more digits is divisible by the integer $\boxed{4}$ if and only if the number formed by its tens and units digits is divisible by $\boxed{4}$ in each member system in this family of base systems.

Proof. For $\xi = 1$ we have the standard result [**1**, p. 94]. Prove by the application of the principle of mathematical induction on the variable "ξ". □

Theorem 15. Let $F = \{B = 2\xi\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.

Then any integer is divisible by $\boxed{\xi}$ if and only if its units digit is $\boxed{0}$ or $\boxed{\xi}$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variable "ξ". (For $\xi = 5$ we have the standard result [**1**, p. 94].) $\square$

Theorem 16. Let $F = \{B\ [B = 2, 3, \cdots]\}$ be a family of base systems.
Then the units digit of $\boxed{B\omega + 1}^{\tau}$ $[\omega = 0, 1, 2, \cdots]$ $[\tau \geq 1]$ is always, $\boxed{1}$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variables "B" and "τ". $\square$

Theorem 17. Let $F = \{B = 2\xi\ [\xi = 2, 3, \cdots]\}$ be a family of base systems.
Then the units digit of $\boxed{2\xi\omega + 2}^{\tau}$ $[\omega = 0, 1, 2, \cdots]$ $[\tau \geq 1]$ is always, $\boxed{2\delta}$. $[\delta = 1, 2, \cdots (\xi - 1)]$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variables "ξ" and "ω" and "τ". $\square$

Theorem 18. Let $F = \{B = 10\xi\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then the units digit of $\boxed{10\xi\omega + 3}^{\tau}$ $[\omega = 0, 1, 2, \cdots]$ $[\tau \geq 1]$ is always,

$\boxed{10\delta+1}$, $\boxed{10\delta+3}$, $\boxed{10\delta+7}$ or $\boxed{10\delta+9}$ $[\delta = 0, 1, 2, \cdots (\xi - 1)]$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variables "ξ" and "ω" and "τ". □

Theorem 19. Let $F = \{B = 2(2\xi + 1)\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then the units digit of $\boxed{2\xi}^{\tau}$ $[\tau \geq 1]$ is always, $\boxed{2\xi}$ or $\boxed{2(\xi+1)}$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variables "ξ" and "τ". □

Theorem 20. Let $F = \{B = 2(2\xi + 1)\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then the units digit of $\boxed{2(2\xi+1)\omega + 2\xi}^{\tau}$ $[\omega = 0, 1, 2, \cdots]$ $[\tau \geq 1]$ is always, $\boxed{2\xi}$ or $\boxed{2(\xi+1)}$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variables "ξ" and "τ". □

Theorem 21. Let $F = \{B = 4\xi\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then the units digit of $\boxed{4\xi\omega + 2\xi}^{\tau}$ $[\omega = 0, 1, 2, \cdots]$ $[\tau \geq 1]$ is always, $\boxed{0}$ or $\boxed{2\xi}$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variables "ξ" and "ω" and "τ". $\square$

Theorem 22. Let $F = \{B = 2(2\xi + 1)\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then the units digit of $\boxed{2(2\xi + 1)\omega + 2\xi + 1}^{\tau}$ $[\omega = 0, 1, 2, \cdots]$ $[\tau \geq 1]$ $\boxed{2\xi + 1}$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variables "ξ" and "ω" and "τ". $\square$

Theorem 23. Let $F = \{B = 2(2\xi + 1)\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then the units digit of $\boxed{2(\xi + 1)}^{\tau}$ $[\tau \geq 1]$ is always, $\boxed{2(\xi + 1)}$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variables "ξ" and "τ". $\square$

Theorem 24. Let $F = \{B = 2(2\xi + 1)\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then the units digit of $\boxed{2(2\xi + 1)\omega + 2(\xi + 1)}^{\tau}$ $[\omega = 0, 1, 2, \cdots]$ $[\tau \geq 1]$ is always, $\boxed{2(\xi + 1)}$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variables "ξ" and "ω" and "τ". $\square$

Theorem 25. Let $F = \{B = 10\xi \ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then the units digit of $\boxed{10\xi\omega + 7}^{\tau}$ $[\omega = 0, 1, 2, \cdots]$ $[\tau \geq 1]$ is always, $\boxed{10\delta + 1}$, $\boxed{10\delta + 3}$, $\boxed{10\delta + 7}$ or $\boxed{10\delta + 9}$ $[\delta = 0, 1, 2, \cdots (\xi - 1)]$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variables "ξ" and "ω" and "τ". □

Theorem 26. Let $F = \{B = 10\xi \ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then the units digit of $\boxed{10\xi\omega + 8}^{\tau}$ $[\omega = 0, 1, 2, \cdots]$ $[\tau \geq 1]$ is always, $\boxed{10\delta + 2}$, $\boxed{10\delta + 4}$, $\boxed{10\delta + 6}$ or $\boxed{10\delta + 8}$. $[\delta = 0, 1, 2, \cdots (\xi - 1)]$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variables "ξ" and "ω" and "τ". □

Theorem 27. Let $F = \{B \ [B = 2, 3, \cdots]\}$ be a family of base systems.
Then the units digit of $\boxed{B\omega + B - 1}^{\tau}$ $[\omega = 0, 1, 2, \cdots]$ $[\tau \geq 1]$ is always, $\boxed{1}$ or $\boxed{B - 1}$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variables "B" and "ω" and "τ". □

Theorem 28. Let $F = \{B = 10\xi\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then for any integer $a^2 - a + \boxed{7}$ the units digit is $\boxed{10\delta + 3}$, $\boxed{10\delta + 7}$ or $\boxed{10\delta + 9}$. $[\delta = 0, 1, 2, \cdots(\xi - 1)]$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variable "ξ". (For $\xi = 1$ we have the standard result [**1**, p. 94].) □

Theorem 29. Let $F = \{B = 10\xi\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then for any integer $a^2 - a + \boxed{10\delta + 7}$ the units digit is $\boxed{10\delta + 3}$, $\boxed{10\delta + 7}$ or $\boxed{10\delta + 9}$. $[\delta = 0, 1, 2, \cdots(\xi - 1)]$ in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variable "ξ". □

Theorem 30. Let $F = \{B\ [B = 2, 3, \cdots]\}$ be a family of base systems.
If T_N denotes the N^{th} triangular number, then $T_{N+2\lambda} \equiv T_N (\text{mod } \lambda)$; hence, T_N and T_{N+2B} must have the same last digit where B is any base in which the triangular numbers are notated.

Proof. Prove by the application of the principle of mathematical induction on the variables "N" and "B". (For $B = 10$ we have the standard result [**1**, p. 95].) □

Theorem 31. Let $F = \{B = 2(2\xi + 1)\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
2^n divides an integer N if and only if 2^n divides the number made up of the last n digits of N in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variable "ξ".
(Hint: $[2(2\xi+1)]^k = 2^k(2\xi+1)^k \equiv 0(\text{mod } 2^n)$ for $k \geq n$ and for all ξ.)
(For $\xi = 2$ we have the standard result [**1**, p. 95].) □

Theorem 32. Let $F = \{B = 2(2\xi + 1) + (2\xi + 1)\omega\ [\xi = 1, 2, \cdots]\ [\omega = 0, 1, 3, 5, \cdots]\}$ be a family of base systems.
$(\omega+2)^n$ divides an integer N if and only if $(\omega+2)^n$ divides the number made up of the last n digits of N in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variables "ξ" and "ω".
(Hint: $[2(2\xi+1)+(2\xi+1)\omega]^k = (\omega+2)^k(2\xi+1)^k \equiv 0(\text{mod } (\omega+2)^n)$ for $k \geq n$ and for all ξ and ω.)
(For $\xi = 2$ and $\omega = 0$ we have the standard result [**1**, p. 95].) □

Theorem 33. Let $F = \{B\ [B = 2, 3, \cdots]\}$ be a family of base systems.
Then for any integer $N > 1$, $\exists$ (there exists) a prime number with at least N of its digits equal to zero in each member system of this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variable "B".
(Hint: Consider the family of arithmetic progressions $B^{N+1}\lambda + 1$ for $\lambda = 1, 2, \cdots$ for each base B.)
(For $B = 10$ we have the standard result [**1**, p. 95].) □

Theorem 34. Let $F = \{B = 3\xi + 1\ [\xi = 3, 4, \cdots]\}$ be a family of base systems.
Let $N = a_m B^m + a_{m-1}B^{m-1} + \cdots + a_1 B + a_0$ be the representation of the positive integer N to the base B, $0 \leq a_k < B$.
Then $\boxed{2\xi + 1}$, $\boxed{3\xi + 2}$ and $\boxed{4\xi + 1}$ all divide N if and only if $\boxed{2\xi + 1}$, $\boxed{3\xi + 2}$ and $\boxed{4\xi + 1}$ divide the integer

$$\begin{aligned} M = &\big[(3\xi + 1)^2 a_2 + (3\xi + 1)a_1 + a_0\big] \\ &- \big[(3\xi + 1)^2 a_5 + (3\xi + 1)a_4 + a_3\big] \\ &+ \big[(3\xi + 1)^2 a_8 + (3\xi + 1)a_7 + a_6\big] - \cdots \end{aligned}$$

in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variable "ξ".
(**Hint:** If n is even, then

$$\begin{aligned} (3\xi + 1)^{3n} &\equiv 1(\text{mod } (3\xi + 1)^3 + 1); \\ (3\xi + 1)^{3n+1} &\equiv (3\xi + 1)(\text{mod } (3\xi + 1)^3 + 1); \\ (3\xi + 1)^{3n+2} &\equiv (3\xi + 1)^2(\text{mod } (3\xi + 1)^3 + 1). \end{aligned}$$

If n is odd, then

$$(3\xi+1)^{3n} \equiv -1(\text{mod } (3\xi+1)^3+1);$$
$$(3\xi+1)^{3n+1} \equiv -(3\xi+1)(\text{mod } (3\xi+1)^3+1);$$
$$(3\xi+1)^{3n+2} \equiv -(3\xi+1)^2(\text{mod } (3\xi+1)^3+1).$$

)

(For $\xi = 3$ we have the standard result [**1**, p. 95].) □

Theorem 35. Let $F = \{B = 2(2\xi+1)\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Let $N = a_m B^m + a_{m-1} B^{m-1} + \cdots + a_1 B + a_0$ be the representation of the positive integer N to the base B, $0 \le a_k < B$.
Then $\boxed{2\xi+1}$ divides N if and only if $\boxed{2\xi+1}$ divides the integer

$$M = a_0 + \boxed{2\xi} a_1 + \boxed{2\xi} a_2 + \cdots + \boxed{2\xi} a_m.$$

in each member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variable "ξ".
(For $\xi = 2$ we have the standard result [**1**, p. 95].) □

Theorem 36. Let $F = \{B = 2(2\xi+1)\ [\xi = 1, 2, \cdots]\}$ be a family of base systems.
Then for any integer N, N and $N^{(2\xi+1)}$ $[\xi = 1, 2, \cdots]$ have the same unit's digit in each corresponding member system in this family of base systems.

Proof. Prove by the application of the principle of mathematical induction on the variable "ξ".
(For $\xi = 2$ we have the standard result [**1**, p.119].) ☐

Theorem 37. Given an integer N, let M be the *Reversido integer*[1] formed by reversing the order of the digits of N in that base system B $[B \geq 2]$.
$(N - M)$ is divisible by $(B - 1)$ in each of these base systems.

Proof. Prove by the application of the principle of mathematical induction on the variable "B".
(For $B = 10$ we have the standard result [**1**, p.95].) ☐

Theorem 38. A *palindrome* is a number N that reads the same backwards as forwards in any base system B $[B \geq 2]$. i.e., When the *Reversido integer* M of an integer N is equal to itself it is *palindrome*.
Any *palindrome* in any base system B $[B \geq 2]$ with an even number of digits is divisible by $(B + 1)$ in that base system.

Proof. Prove by the application of the principle of mathematical induction on the variable "B".
(For $B = 10$ we have the standard result [**1**, p.95].) ☐

Theorem 39. Given a repunit R_n, represented to any base B $[B \geq 2]$

$(B - 1)|R_n$ if and only if $(B - 1)|n$.

[1] This terminology is new. I hope that it is acceptable!

Proof. Prove by the application of the principle of mathematical induction on the variable "B".
(For $B = 10$ we have the standard result [**1**, p.95].) □

Theorem 40. Given a repunit R_n, represented to any base B

$(B+1)|R_n$ if and only if n is even.

Proof. Prove by the application of the principle of mathematical induction on the variable "B".
(For $B = 10$ we have the standard result [**1**, p.95].) □

Theorem 41. Let I_n be an integer I repeated n times and called a repinteger I_n.
Given a repunit R_n, or more generally a repinteger I_n represented to any base B

$R_m|R_n$ if and only if $m|n$. [for all $m > 1$ and $n > 1$]

$I_m|I_n$ if and only if $m|n$. [for all $m > 1$ and $n > 1$]

Proof. Trivial! □

Theorem 42. Let $\overset{\infty}{I}$ be repinteger-infinity i.e., an integer I repeated to infinity. i.e., $IIIIIIIIIIIIIIIIIIIIIIIIII \cdots \infty$ $[I = 1, 2, \cdots \infty]$

Every repinteger-infinity $\overset{\infty}{I}$ is divisible by all the factors f of the repeating integer I and it yields a repinteger-infinity $\overset{\infty}{J}$ with J having the same number of digits as I. When the number of digits of J is less than I then the vacant digit places are filled with zeroes.

i.e.,

$$\frac{\overset{\infty}{I}}{f} = \overset{\infty}{J} \quad \text{and} \quad \frac{\overset{\infty}{I}}{\overset{\infty}{J}} = f.$$

For eg:

$$\overset{\infty}{128} = 128128128128128 \cdots\cdots \infty \text{ divided by 16 yields}$$

$$008008008008008 \cdots\cdots \infty = \overset{\infty}{008}$$

i.e.,

$$\frac{\overset{\infty}{128}}{\overset{\infty}{008}} = 16.$$

Theorem 43.

a). A repinteger I_n $[I > 1]$ and a repinteger-infinity $\overset{\infty}{I}$ $[I > 1]$ represented in any base B are composite.

b). Let $[\overset{\infty}{kx}]$ be the linearly rhythmic infinitely long integer $[lrili]$ $[\overset{\infty}{kx}] = kx, 2kx, 3kx, \cdots\infty$ and let $[x]_n = 1kx, 2kx, 3kx, \cdots, nkx$ for all $k > 1$ and $x \geq 1$. For eg:

$$[\overset{\infty}{2.7}] = 714212835424956 \cdots\cdots \infty$$

$$[\overset{\infty}{5.7}] = 73570105140175 \cdots\cdots \infty$$

for all $k > 1$ and $x \geq 1$, k is a factor of $[\overset{\infty}{kx}]$ and $[kx]_n$ and hence $[\overset{\infty}{kx}]$ and $[kx]_n$ represented in any base B are composite.

c). Infinite possible functional zero-rhythms $[fzor]$ {in fact any number of zeroes in any random fashion finite or infinite} may be inducted between the repeating integers in any repinteger-infinity $[repi]$ $\overset{\infty}{I}$ $[I > 1]$

and between the rhythmic integers in the sequences of any linearly rhythmic infinitely long integer $[lrili]$ $[\overset{\infty}{kx}] = kx, 2kx, 3kx, \cdots \infty$ for all $k > 1$ and $x \geq 1$.

All $[fzor/lrili]$ and $[fzor/repi]$ represented in any base B are composite. For eg:

1. $3/[\overset{\infty}{kx}] = kx, 0_3 2kx, 0_3 3kx, 0_3 \cdots \infty$ for all $k > 1$ and $x \geq 1$

2. $F/[\overset{\infty}{kx}] = kx, 0_{F(1)} 2kx, 0_{F(2)} 3kx, 0_{F(3)} \cdots \infty$ for all $k > 1$ and $x \geq 1$.

[Here $F(\tau)$ is any function of a random variable τ. Many-valued functions and any generalized function may also be used to generate the zero rhythms.]

Notes: Any infinitely long natural number N_∞ [limit integer] which is not a A repinteger-infinity $[repi]$ $\overset{\infty}{I}$ $[I > 1]$ or a linearly rhythmic infinitely long integer $[lrili]$ $[[\overset{\infty}{kx}]]$ (for all $k > 1$ and $x \geq 1$) or $[fzor/lrili]$ and $[fzor/repi]$ is a limit prime number.

Notes $\sim$

Any infinitely long natural number N_∞ [limit integer] which has no finite factor other than 1 is a limit prime number.

i.e.,

Any infinitely long natural number N_∞ [limit integer] which is composite has at least one finite factor other than 1.

References

[1] David M. Burton; *Elementary Number Theory*, Universal Book Stall, New Delhi, Second Edition. Reprint 1998.